

# A note on the supersymplectic structure of triplectic formalism

Bodo Geyer<sup>a,\*</sup>, Peter Lavrov<sup>a,b,†</sup> and Armen Nersessian<sup>a,c,d,‡</sup>

<sup>a)</sup> *Center of Theoretical Studies, Leipzig University, Augustusplatz 10/11, D-04109 Leipzig, Germany*

<sup>b)</sup> *Tomsk State Pedagogical University, 634041 Tomsk, Russia*

<sup>c)</sup> *Yerevan State University, A. Manoogian St.,1, Yerevan, 375025, Armenia*

<sup>d)</sup> *Yerevan Physics Institute, Alikhanian Brothers St.,2, Yerevan, 375036, Armenia*

## Abstract

We equip the whole space of fields of the triplectic formalism of Lagrangian quantization with an even supersymplectic structure and clarify its geometric meaning. We also discuss its relation to a closed two-form arising naturally in the superfield approach to the triplectic formalism.

## 1 Introduction

The Batalin-Vilkovisky (BV) formalism [1] of Lagrangian quantization of general gauge theories, since its introduction, attracts permanent interest due to its covariance, universality and mathematical elegance. Now, its area of physical applications is much wider than offered in the initial prescription. The BV-formalism is outstanding also from the mathematical point of view because it is formulated in terms of seemingly exotic objects: the antibracket (odd Poisson bracket) and the related second-order operator  $\Delta$ .

The study of the geometric structure of the BV formalism allowed to introduce its interpretation in terms of more traditional mathematical objects [2]. On the other hand, there exists a more complicated,  $Sp(2)$  symmetric extension of the BV formalism [3], and of its geometrized version known as ‘triplectic formalism’ [4, 5] (see also [6]).

In the BV formalism the original set of ‘physical’ fields  $x^i, \epsilon(x^i) \equiv \epsilon_i$  (including ghosts, antighosts, Lagrangian multipliers etc.), is doubled by the ‘antifields’  $\theta_i$  with opposite grading. On this set of fields and antifields the *nondegenerate* antibracket and the corresponding  $\Delta$ -operator are defined. Differently, the triplectic formalism deals with two sets of auxiliary fields  $\theta_{ai}$ ,  $a = 1, 2$ , which could be arranged in the set of triplets  $(x^i, \theta_{ai}), \epsilon(\theta_{ai}) = \epsilon_i + 1$ . The fields  $x^i$  parametrize the subspace  $\mathcal{M}_0$  endowed with an *even* (super)symplectic structure,

$$\omega = \omega_{ij}(x) dx^j \wedge dx^i, \quad d\omega = \omega_{ij,k} dx^k \wedge dx^j \wedge dx^i = 0. \quad (1)$$

The components  $\omega_{ij}$  obey the relations

$$\omega_{ij} = -(-1)^{\epsilon_i \epsilon_j} \omega_{ji}, \quad (-1)^{\epsilon_j \epsilon_k} \omega_{ki,j} + (-1)^{\epsilon_i \epsilon_j} \omega_{jk,i} + (-1)^{\epsilon_i \epsilon_k} \omega_{ij,k} = 0, \quad \epsilon(\omega_{ij}) = \epsilon_i + \epsilon_j. \quad (2)$$

The inverse tensor  $\omega^{ij}, \omega^{ik} \omega_{kj} (-1)^{\epsilon_k} = \delta_j^i$ , defines on  $\mathcal{M}_0$  an even Poisson bracket,

$$\{f(x), g(x)\}_0 = \frac{\partial_r f}{\partial x^i} \omega^{ij} \frac{\partial_l g}{\partial x^j}. \quad (3)$$

The whole space of fields and antifields,  $\mathcal{M}$ , is equipped with a pair of *degenerate* antibrackets,

$$(f(x, \theta), g(x, \theta))^a = \frac{\partial_l f}{\partial x^i} \frac{\partial_r g}{\partial \theta_{ai}} - \frac{\partial_l f}{\partial \theta_{ai}} \frac{\partial_r g}{\partial x^i}, \quad (4)$$

together with a related pair of operators  $\Delta^a$ ; also some additional odd vector fields  $V^a$  are needed which, in some special case [4], could be absorbed by the action.

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\*e-mail: geyer@itp.uni-leipzig.de

†e-mail: lavrov@tspu.edu.ru

‡e-mail: nerses@thsun1.jinr.ru

The geometry underlying the triplectic formalism, is quite rich and unusual. There were some efforts to understand it from various points of view [7], as well as to find explicit non-trivial examples of such triplectic spaces  $\mathcal{M}$ . In particular, in our previous papers [8, 9], we tried to give a covariant (coordinate-free) realization of the triplectic formalism, equipping the space  $\mathcal{M}_0$  with a connection which respects the symplectic structure (1). However, we found that the basic relations of the triplectic formalism could be fulfilled in such an approach for flat symplectic connections only. On the other hand, in [8] we found that the implementation of that symplectic connection allows to equip the whole triplectic space  $\mathcal{M}$  with an even symplectic structure which provides the triplectic formalism with a well-defined integration measure.

In the papers [8], considering an even symplectic structure, we restricted ourselves to the case of flat connections. We also assumed, for the sake of simplicity, that the initial space  $\mathcal{M}_0$  is a purely bosonic one. Of course, in the triplectic formalism,  $\mathcal{M}_0$  is necessarily a supermanifold (containing, besides the original gauge fields, also ghosts and antighosts as well as matter fields). In the paper [9] we repaired this, but gave no detailed study of the (even) supersymplectic structure and its geometric implications.

In this note, avoiding the just mentioned restrictions, we equip the whole space  $\mathcal{M}$  of the triplectic formalism with an even supersymplectic structure  $\Omega$  and we clarify their geometric origin. With the aim to lay the ground for finding a realization of the triplectic algebra also in that general non-flat case we consider a closed two-form arising naturally in the superfield formulation of triplectic formalism [10] and try to relate it to  $\Omega$ . This, however, seems not to be straightforward and should be re-considered.

## 2 Even (Super) Symplectic Structures

First, we repeat the introduction of an even symplectic structure in the restricted case of a (bosonic) manifold  $\mathcal{M}_0$  but without assuming its flatness. After that, we give a concise geometric formulation of constructing that symplectic structure which, then, will be generalized to the case of a supermanifold  $\mathcal{M}_0$ .

When the Poisson bracket (3) is non-degenerate, the superspace  $\mathcal{M}$  can be equipped with both an *even symplectic structure* and a corresponding *non-degenerate Poisson bracket* analogous to [8],

$$\Omega = \omega + \alpha^{-1} d(\theta^{ia} \omega_{ij} D\theta_a^j) = \left( \omega_{ij} + \frac{1}{2\alpha} R_{ijkl} \theta^{ka} \theta_a^l \right) dx^i \wedge dx^j + \frac{1}{\alpha} \omega_{ij} D\theta^{ia} \wedge D\theta_a^j, \quad (5)$$

where  $\alpha$  is an arbitrary constant, the  $Sp(2)$ -indices are lowered by the invariant  $Sp(2)$ -tensor  $\epsilon_{ab}$ ,  $\theta_a^i = \epsilon_{ab} \theta^{ib}$ , and the covariant derivative is defined by  $D\theta^{ia} = d\theta^{ia} + \Gamma_{kl}^i \theta_k^a dx^l$ ; thereby,  $\Gamma_{kl}^i(x)$  are the coefficients of the connection which respects the symplectic structure:

$$\partial_k \omega_{ij} - \Gamma_{ki}^l \omega_{lj} - \omega_{il} \Gamma_{kj}^l = 0, \quad (6)$$

while  $R_{ijkl} = \omega_{im} R_{jkl}^m$ , with  $R_{jkl}^i$  being the curvature components of that symplectic connection,

$$R_{jkl}^i = -\Gamma_{kj,l}^i + \Gamma_{lj,k}^i + \Gamma_{km}^i \Gamma_{lj}^m - \Gamma_{lm}^i \Gamma_{kj}^m. \quad (7)$$

The indices  $i$  are lowered by the help of the symplectic structure, e.g.  $\theta_{ia} = \omega_{ij} \theta_a^j$ .

Obviously, the suggested symplectic structure  $\Omega$  transforms covariant under the following change of coordinates,

$$\bar{x}^i = \bar{x}^i(x), \quad \bar{\theta}_a^i = \frac{\partial \bar{x}^i}{\partial x^j} \theta_a^j, \quad (8)$$

so that  $\theta_a^i$  could be identified with (two different) one-forms  $dx^i$ . It is clear that, due to the presence of the  $Sp(2)$  indices, it is possible to describe not only external forms on  $\mathcal{M}_0$ , i.e. antisymmetric covariant tensors, but also specific symmetric ones as well. As is well-known, one could equip the space  $\mathcal{M}$  with a pair of antibrackets (4) which transform covariant under the coordinate changes (8).

Furthermore, using the symplectic structure (5) we can introduce the following nondegenerate Poisson bracket, thereby extending the Poisson bracket (4) from  $\mathcal{M}_0$  to the whole space  $\mathcal{M}$ ,

$$\{f(z), g(z)\} = (\nabla_i f) \tilde{\omega}^{ij} (\nabla_j g) + \alpha \frac{\partial_r f}{\partial \theta_a^i} \omega^{ij} \frac{\partial_l g}{\partial \theta^{ja}}; \quad (9)$$

here, we used the notations

$$\tilde{\omega}^{im}\tilde{\omega}_{mj} \equiv \tilde{\omega}^{im}(\omega_{mj} + \frac{1}{2\alpha} R_{mjk} \theta^{ka} \theta_a^l) = \delta_j^i, \quad (10)$$

$$\nabla_i \equiv \frac{\partial}{\partial x^i} - \Gamma_{ij}^k(x) \theta^{ja} \frac{\partial}{\partial \theta^{ka}} \quad \text{with} \quad [\nabla_i, \nabla_j] = R_{lij}^k \theta^{la} \frac{\partial}{\partial \theta_a^k}. \quad (11)$$

Obviously, the operator  $\nabla_k$  acts on the monomials  $\alpha_{a_1 \dots a_n} = \alpha_{[i_1 \dots i_n]} \theta_{a_1}^{i_1} \dots \theta_{a_n}^{i_n}$  as a covariant derivative

$$\nabla_k \alpha_{a_1 \dots a_n} = \alpha_{[i_1 \dots i_n; k]} \theta_{a_1}^{i_1} \dots \theta_{a_n}^{i_n}. \quad (12)$$

The symplectic structure  $\Omega$  which we introduced above has a simple geometrical meaning. To show this, let us remind some general procedure for introducing a supersymplectic structure:

(1) Let us consider some supermanifold  $\mathcal{M}$  being given as the vector bundle of some symplectic manifold  $\mathcal{M}_0$ . Furthermore, let  $\theta^\mu$  be odd coordinates parametrizing the fibers of that bundle, and let  $x^i$  be local coordinates of the base manifold  $\mathcal{M}_0$ . Let  $g_{\mu\nu} = g_{\mu\nu}(x)$  be a metric on the bundle, and  $\Gamma_{i\nu}^\mu$  be the components of its connection, so that

$$g_{\mu\nu} = g_{\nu\mu}, \quad g_{\mu\nu;k} = g_{\mu\nu,k} - g_{\mu\alpha} \Gamma_{k\nu}^\alpha - g_{\alpha\nu} \Gamma_{k\mu}^\alpha = 0. \quad (13)$$

On such a supermanifold we can define a symplectic structure as follows:

$$\begin{aligned} \Omega &= \omega + \frac{1}{\alpha} d(\theta^\mu g_{\mu\nu}(x) \mathcal{D}\theta^\nu) \\ &= \omega + \frac{1}{2\alpha} R_{\nu\mu ki} \theta^\nu \theta^\mu dx^i \wedge dx^k + \frac{1}{\alpha} g_{\mu\nu} \mathcal{D}\theta^\nu \wedge \mathcal{D}\theta^\mu, \end{aligned} \quad (14)$$

where  $\mathcal{D}\theta^\mu = d\theta^\mu + \Gamma_{\nu i}^\mu \theta^\nu dx^i$ ,  $R_{\mu\nu ki} = g_{\mu\alpha} R_{\nu ki}^\alpha$ , while  $R_{\nu ki}^\mu$  are the curvature components of the connection.

Now, let us specify this symplectic structure to our case, i.e., let us choose  $\theta^\mu = \theta^{ia}$ . In this specification  $\mu, \nu$  are multi-indices:  $\mu = (i, a)$ ,  $\nu = (j, b)$ , and we choose the following metric and connection:

$$g_{\mu\nu} = \omega_{ij} \epsilon_{ab}, \quad \Gamma_{\nu i}^\mu = \Gamma_{jk}^i \delta_b^a. \quad (15)$$

Upon such specification, from the covariant constancy of the metric  $g_{\mu\nu}$ , Eq. (13), it immediately follows that  $\Gamma_{jk}^i$  is a symplectic connection on  $\mathcal{M}_0$ . The curvature of that connection is also reduced to the curvature of the symplectic connection,

$$R_{\mu\nu kl} = g_{\mu\alpha} R_{\nu kl}^\alpha = \epsilon_{ab} \omega_{im} R_{jkl}^m = \epsilon_{ab} R_{ijkl}, \quad R_{jkl}^i = -\Gamma_{kj,l}^i + \Gamma_{lj,k}^i + \Gamma_{km}^i \Gamma_{lj}^m - \Gamma_{lm}^i \Gamma_{kj}^m. \quad (16)$$

Hence, we get precise correspondence with the symplectic structure (5).

(2) It is easy to extend the above construction to the case when  $\mathcal{M}_0$  is an even symplectic *supermanifold* with local coordinates  $x^i$ ,  $\epsilon(x^i) \equiv \epsilon_i$ . We shall follow De Witt's definitions and conventions concerning tensor fields on supermanifolds [11] (see also [9]). In particular, if the sets  $\{e_i = \frac{\partial}{\partial x^i}\}$  and  $\{e^i = dx^i\}$  are coordinate bases in the tangent and the cotangent spaces, respectively, then they transform under a change of local coordinates  $x^i \rightarrow \bar{x}^i = \bar{x}^i(x)$  according to the rules

$$\bar{e}_i = e_j \frac{\partial x^j}{\partial \bar{x}^i}, \quad \bar{e}^i = e^j \frac{\partial \bar{x}^i}{\partial x^j}. \quad (17)$$

These vectors are dual with respect to an inner product operation,  $\langle \cdot, \cdot \rangle$ ,

$$\langle e^i, e_j \rangle = \delta_j^i, \quad \langle e_j, e^i \rangle = (-1)^{\epsilon_i} \delta_j^i, \quad (18)$$

obeying the following properties:

$$\langle \omega, X_1 + X_2 \rangle = \langle \omega, X_1 \rangle + \langle \omega, X_2 \rangle, \quad \langle \omega, X \rangle = \langle X, \omega \rangle (-1)^{\epsilon(\omega)\epsilon(X)}, \quad (19)$$

and

$$\langle \omega, X_1 X_2 \rangle = \langle \omega, X_1 \rangle X_2 + \langle \omega, X_2 \rangle X_1 (-1)^{\epsilon(X_1)\epsilon(X_2)}. \quad (20)$$

The coordinates parametrising the fibers,  $\theta^\mu = \theta^{i\alpha}$ , could also be even and odd:  $\epsilon(\theta^\mu) = \epsilon_\mu + 1 = \epsilon_i + 1$ . In that case the analog of the supersymplectic structure (14) reads

$$\begin{aligned}\Omega &= \omega + \frac{1}{\alpha} d(g_{\mu\nu}(x)\theta^\nu \mathcal{D}\theta^\mu (-1)^{\epsilon_\nu}) \\ &= \omega - \frac{1}{2\alpha} g_{\mu\alpha} R^\alpha_{\nu ki} dx^i \wedge dx^k \theta^\nu \theta^\mu (-1)^{\epsilon_\nu} + \frac{1}{\alpha} g_{\mu\nu} \mathcal{D}\theta^\nu \wedge \mathcal{D}\theta^\mu (-1)^{\epsilon_\nu},\end{aligned}\quad (21)$$

where

$$g_{\mu\nu} = (-1)^{\epsilon_\nu \epsilon_\mu} g_{\nu\mu}, \quad \mathcal{D}\theta^\nu = d\theta^\nu + \Gamma^\nu_{i\lambda}(x) \theta^\lambda dx^i (-1)^{\epsilon_\nu + \epsilon_\lambda + \epsilon_i}, \quad \epsilon(g_{\mu\nu}) = \epsilon_\mu + \epsilon_\nu, \quad (22)$$

while the curvature tensor is defined as follows

$$R^\nu_{\alpha ki} = -\Gamma^\nu_{k\alpha, i} (-1)^{\epsilon_k \epsilon_\alpha} + \Gamma^\nu_{i\alpha, k} (-1)^{\epsilon_i (\epsilon_k + \epsilon_\alpha)} + \Gamma^\nu_{k\beta} \Gamma^\beta_{i\alpha} (-1)^{(\epsilon_k + \epsilon_i) \epsilon_\alpha} - \Gamma^\nu_{i\beta} \Gamma^\beta_{k\alpha} (-1)^{(\epsilon_i + \epsilon_k) \epsilon_\alpha + \epsilon_i \epsilon_k}. \quad (23)$$

It is antisymmetric w.r.t. the last two indices:  $R^\nu_{\alpha ik} = -(-1)^{\epsilon_i \epsilon_k} R^\nu_{\alpha ki}$ .

The connection  $\Gamma^\mu_{k\nu}$  respects the metric  $g_{\mu\nu}$ ,

$$g_{\mu\nu; k} = g_{\mu\nu, k} - g_{\mu\alpha} \Gamma^\alpha_{k\nu} (-1)^{\epsilon_k \epsilon_\nu} - g_{\alpha\nu} \Gamma^\alpha_{k\mu} (-1)^{\epsilon_\nu (\epsilon_\alpha + \epsilon_\mu) + \epsilon_k \epsilon_\mu} = 0, \quad (24)$$

and, under the change of coordinates,

$$\bar{x}^i = \bar{x}^i(x), \quad \bar{\theta}^\nu = \theta^\mu A_\mu{}^\nu(x), \quad \epsilon(A_\mu{}^\nu) = \epsilon_\mu + \epsilon_\nu, \quad (25)$$

it transforms as follows:

$$\bar{\Gamma}^\mu_{i\nu} = A^\mu{}_\lambda \Gamma^\lambda_{k\alpha} \frac{\partial_r x^k}{\partial \bar{x}^i} B^\alpha{}_\nu (-1)^{\epsilon_\alpha (\epsilon_i + \epsilon_k)} - A^\mu{}_{\alpha, k} B^\alpha{}_\nu \frac{\partial_r x^k}{\partial \bar{x}^i} (-1)^{\epsilon_\alpha \epsilon_k + \epsilon_\nu (\epsilon_i + \epsilon_k)}, \quad (26)$$

where

$$A_\mu{}^\nu B_\nu{}^\lambda = B_\mu{}^\nu A_\nu{}^\lambda = \delta_\mu{}^\lambda, \quad A_\mu{}^\nu = A^\nu{}_\mu (-1)^{\epsilon_\mu (\epsilon_\nu + 1)}, \quad B^\mu{}_\nu = B_\nu{}^\mu (-1)^{\epsilon_\nu (\epsilon_\mu + 1)}. \quad (27)$$

Hence,  $\mathcal{D}\theta^\nu$  transforms homogeneous,  $\mathcal{D}\bar{\theta}^\nu = \mathcal{D}\theta^\mu A_\mu{}^\nu(x)$ , under the above change of coordinates (25).

Now, let us choose

$$g_{\mu\nu} = \omega_{ij} \epsilon_{ab}, \quad \Gamma^\nu_{k\mu} = \Gamma^j_{ki} \delta_a^b. \quad (28)$$

Then the condition (24) takes the form

$$g_{\mu\nu; k} = 0 \quad \rightarrow \quad \epsilon_{ab} [\omega_{ij, k} - \omega_{il} \Gamma^l_{kj} (-1)^{\epsilon_k \epsilon_j} + \omega_{jl} \Gamma^l_{ki} (-1)^{\epsilon_i \epsilon_j + \epsilon_i \epsilon_k}] = 0, \quad (29)$$

i.e.,  $\Gamma^j_{ki}$  in (28) defines a symplectic connection on the supermanifold  $\mathcal{M}$ . In this case we have the following representation for curvature tensor (23) [12]:

$$\begin{aligned}R^\alpha_{\nu kl} &= \delta_b^a R^m_{jkl}, \\ R^i_{jkl} &= -\Gamma^i_{kj, l} + \Gamma^i_{lj, k} (-1)^{\epsilon_k \epsilon_l} + \Gamma^i_{km} \Gamma^m_{lj} (-1)^{\epsilon_i + \epsilon_k (\epsilon_j + \epsilon_m)} - \Gamma^i_{lm} \Gamma^m_{kj} (-1)^{\epsilon_i + \epsilon_l (\epsilon_j + \epsilon_m) + \epsilon_l \epsilon_k},\end{aligned}\quad (30)$$

with  $R^i_{jkl}$  being the curvature of the supersymplectic connection.

In case when the connection is flat one is able to find a realization of the triplectic algebra on  $\mathcal{M}$  [12]; for the general case this remains an open question.

### 3 Superfield approach

It seems to be advantageous to attack that problem in the more general superfield approach to  $Sp(2)$  symmetric quantization [10]. In that approach we deal with the superfield  $\phi^i$ ,

$$\phi^i = x^i + \eta^a \theta_a^i + \eta^1 \eta^2 y^i, \quad \text{where} \quad \theta_a^i = (-1)^{\epsilon_i} \omega^{ij} \theta_{ja}, \quad y^i = (-1)^{\epsilon_j} \omega^{ij} y_j, \quad (31)$$

and  $\epsilon(y^i) = \epsilon_i$ ,  $\epsilon(\theta_a^i) = \epsilon_i + 1$ ,  $\epsilon(\eta^a) = 1$ .

Let us naively define, on the superfield space, the symplectic structure  $\omega(\phi) = \omega_{ij}(\phi) d\phi^j \wedge d\phi^i$ . Expanding this form on Grassmann parameters  $\eta^a$ , we obtain

$$\omega(\phi) = \omega_{ij}(x) dx^j \wedge dx^i - 2\eta^a dx^i \wedge d\theta_{ia} - \eta^1 \eta^2 \left[ 2dy_i \wedge dx^i + \tilde{\Omega}(x, \theta) \right], \quad (32)$$

where

$$\tilde{\Omega} = -\frac{1}{2} \tilde{R}_{klij} (-1)^{\epsilon_l \epsilon_{ba}} \theta^{lb} \theta^{ka} dx^j \wedge dx^i + \omega_{ij} \epsilon_{ab} (-1)^{\epsilon_j} \tilde{\mathcal{D}} \theta^{jb} \wedge \tilde{\mathcal{D}} \theta^{ia}. \quad (33)$$

In (33) we used the notation:

$$\begin{aligned} \tilde{R}_{klij} &= \omega_{ij,kl} - \omega_{im,k} \omega^{mn} \omega_{nj,l} (-1)^{\epsilon_i + \epsilon_j + \epsilon_k + \epsilon_l + \epsilon_m + \epsilon_k(\epsilon_m + \epsilon_j)} + \\ &\quad \omega_{jm,k} \omega^{mn} \omega_{in,l} (-1)^{\epsilon_i + \epsilon_j + \epsilon_k + \epsilon_l + \epsilon_m + \epsilon_i \epsilon_j + \epsilon_k(\epsilon_m + \epsilon_i)}, \end{aligned} \quad (34)$$

$$\tilde{\mathcal{D}} \theta^{ia} = d\theta^{ia} + \tilde{\Gamma}_{jk}^i \theta^{ka} dx^j (-1)^{(\epsilon_i + \epsilon_j + \epsilon_k)}, \quad \text{where} \quad \tilde{\Gamma}_{jk}^i = \omega^{ip} \omega_{pj,k} (-1)^{\epsilon_j + \epsilon_k}. \quad (35)$$

Seemingly, the formal structure of  $\tilde{\Omega}$  is the same as for the supersymplectic structure (21) in Section 2. The quantity  $\tilde{R}_{ijkl}$  obeys the following symmetry properties,

$$\tilde{R}_{klij} = -(-1)^{\epsilon_i \epsilon_j} \tilde{R}_{klji}, \quad \tilde{R}_{klij} = (-1)^{\epsilon_k \epsilon_l} \tilde{R}_{lkij}, \quad (36)$$

as it should be for a supersymplectic curvature tensor. Of course, we also have to prove that  $\tilde{\Gamma}$  is a symplectic connection, i.e. respecting  $\omega$ , and that  $\tilde{R}$  may be rewritten in the form (30). However, one can check that  $\tilde{\Gamma}_{jk}^i$  in (35) can not be considered as a symplectic connection satisfying (29).

The superfield symplectic structure  $\omega(\phi)$  transforms covariant under a change of the superfield coordinates  $\bar{\phi}^i = \bar{\phi}^i(\phi)$ :

$$\bar{\omega}(\bar{\phi}) = \omega(\phi), \quad d\bar{\phi}^i = d\phi^j \frac{\partial \bar{\phi}^i}{\partial \phi^j}, \quad \bar{\omega}_{ij}(\bar{\phi}) = \omega_{mn}(\phi) \frac{\partial_r \phi^n}{\partial \bar{\phi}^j} \frac{\partial_r \phi^m}{\partial \bar{\phi}^i} (-1)^{\epsilon_j(\epsilon_i + \epsilon_m)}. \quad (37)$$

In component form the coordinates transform according to

$$\bar{x}^i = \bar{x}^i(x), \quad \bar{\theta}_a^i = \theta_a^j \frac{\partial \bar{x}^i}{\partial x^j}, \quad \bar{y}^i = y^j \frac{\partial \bar{x}^i}{\partial x^j} + \frac{1}{2} \epsilon_{ab} \theta^{ja} \theta^{kb} \frac{\partial^2 \bar{x}^i}{\partial x^k \partial x^j} (-1)^{\epsilon_j}. \quad (38)$$

Hence, the even two-form  $\omega(x)$ , the pair of odd two-forms  $dx^i \wedge d\theta_{ia}$  as well as the two-form  $2dy_i \wedge dx^i + \tilde{\Omega}$  are covariant w.r.t. these transformations. Obviously,  $\Omega$  itself is not covariant under the given transformation, since  $\tilde{\Gamma}_{jk}^i$  is not a connection (and, therefore,  $\tilde{R}_{klij}$  (34) is not a tensor) on  $\mathcal{M}_0$ .

Indeed, let us introduce, as in the previous section, the metric and the connection with components

$$g_{\mu\nu} = \omega_{ij} \epsilon_{ab}, \quad \tilde{\Gamma}_{i\nu}^\mu = \tilde{\Gamma}_{i(k,b)}^{(j,a)} = \tilde{\Gamma}_{ik}^j \delta_b^a, \quad \mu = (i, a), \quad \nu = (j, b). \quad (39)$$

Notice, that  $\tilde{\Gamma}_{jk}^i$  does not respects the metric  $g_{\mu\nu}$ :

$$g_{\mu\nu;k} \neq 0.$$

In principle, it is possible to achieve respecting the metric if we change the definition of  $\tilde{\Gamma}_{jk}^i$  in (33) by omitting the factor  $(-1)^{(\epsilon_i + \epsilon_j + \epsilon_k)}$  in  $\tilde{\mathcal{D}} \theta^{ia}$ . But in any case  $\tilde{\Gamma}_{jk}^i$  does not transform, under a change of local coordinates of the base supermanifold  $\mathcal{M}_0$ ,  $(x) \rightarrow (\bar{x})$ , as a connection

$$\bar{\Gamma}_{jk}^i \neq \frac{\partial_r \bar{x}^i}{\partial x^p} \Gamma_{mn}^p \frac{\partial_r x^n}{\partial \bar{x}^k} \frac{\partial_r x^m}{\partial \bar{x}^j} (-1)^{\epsilon_k(\epsilon_j + \epsilon_m)} + \frac{\partial_r \bar{x}^i}{\partial x^p} \frac{\partial_r^2 x^p}{\partial \bar{x}^j \partial \bar{x}^k}. \quad (40)$$

Hence,  $\tilde{\Gamma}_{jk}^i$  could not be interpreted as a connection on  $\mathcal{M}_0$ ! Similarly,  $\tilde{R}_{ijkl}$  could not be considered as a curvature of the connection on  $\mathcal{M}_0$ . This explains, why the two-form  $\Omega$  is not covariant under the transformation  $\bar{x}^i = \bar{x}^i(x)$ ,  $\bar{\theta}_a^i = \theta_a^j \frac{\partial \bar{x}^i}{\partial x^j}$ .

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